NAG Toolbox for MATLAB

f07fr

1 Purpose

f07fr computes the Cholesky factorization of a complex Hermitian positive-definite matrix.

2 Syntax

```
[a, info] = f07fr(uplo, a, 'n', n)
```

3 Description

f07fr forms the Cholesky factorization of a complex Hermitian positive-definite matrix A either as $A = U^{\mathrm{H}}U$ if **uplo** = 'U' or $A = LL^{\mathrm{H}}$ if **uplo** = 'L', where U is an upper triangular matrix and L is lower triangular.

4 References

Demmel J W 1989 On floating-point errors in Cholesky *LAPACK Working Note No. 14* University of Tennessee, Knoxville

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo – string**

Indicates whether the upper or lower triangular part of A is stored and how A is to be factorized.

uplo = 'U'

The upper triangular part of A is stored and A is factorized as $U^{\mathrm{H}}U$, where U is upper triangular.

uplo = 'L'

The lower triangular part of A is stored and A is factorized as LL^{H}, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **a**(**lda**,$*$) **– complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The n by n Hermitian positive-definite matrix A.

If **uplo** = 'U', the upper triangular part of A must be stored and the elements of the array below the diagonal are not referenced.

If **uplo** = 'L', the lower triangular part of A must be stored and the elements of the array above the diagonal are not referenced.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The second dimension of the array **a**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda

5.4 Output Parameters

1: **a(lda,∗) – complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The upper or lower triangle of A contains the Cholesky factor U or L as specified by **uplo**.

2: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **a**, 4: **lda**, 5: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** $= i$, the leading minor of order i is not positive-definite and the factorization could not be completed. Hence A itself is not positive-definite. This may indicate an error in forming the matrix A. To factorize a Hermitian matrix which is not positive-definite, call f07mr instead.

7 Accuracy

If **uplo** $=$ 'U', the computed factor U is the exact factor of a perturbed matrix $A + E$, where

$$|E| \leq c(n)\epsilon \left|U^{\mathrm{H}}\right||U|,$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***. If **uplo** $=$ 'L', a similar statement holds for the computed factor L. It follows that $\left|e_{ij}\right| \leq c(n)\epsilon\sqrt{a_{ii}a_{jj}}$.

8 Further Comments

The total number of real floating-point operations is approximately $\frac{4}{3}n^3$.

A call to f07fr may be followed by calls to the functions:

f07fs to solve $AX = B$;

f07fu to estimate the condition number of A;

f07fw to compute the inverse of A.

The real analogue of this function is f07fd.

9 Example

```
uplo = 'L';
a = [complex(3.23, +0), complex(0, 0), complex(0, 0), complex(0, 0);
       complex(1.51, +1.92), complex(3.58, 0), complex(0, 0), complex(0,
0);
        complex(1.9, -0.84), complex(-0.23, -1.11), complex(4.09, +0),
complex(0, 0);
       complex(0.42, -2.5), complex(-1.18, -1.37), complex(2.33, +0.14),
complex(4.29, +0)];
[aOut, info] = f07fr(uplo, a)
```

```
aOut =
   1.7972                  0                  0                  0
   0.8402 + 1.0683i   1.3164                  0                  0
   1.0572 - 0.4674i  -0.4702 + 0.3131i   1.5604                  0
   0.2337 - 1.3910i   0.0834 + 0.0368i   0.9360 + 0.9900i   0.6603
info =
           0
```